OCR
Oxford Cambridge and RSA

Friday 23 June 2017 – Morning

A2 GCE MATHEMATICS

4724/01 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 **(i)** Find the first three terms in ascending powers of x in the binomial expansion of $\sqrt[4]{1+8x}$. **[3]**

(ii) State the range of values for which this expansion is valid. **[1]**

2 The equations of two lines are

$$\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} -1 \\ 8 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix}.$$

Find the coordinates of the point where these lines intersect. **[4]**

3 Show that $\int_0^1 16x\mathrm{e}^{4x}\,\mathrm{d}x = 3\mathrm{e}^4 + 1$. **[5]**

4 Express $\dfrac{9x^2+43x+8}{(3+x)(1-x)(2x+1)}$ in partial fractions. **[5]**

5 **(i)** Find the quotient and the remainder when $6x^4+12x^3-3x^2-11x-2$ is divided by $2x^2+4x+1$. **[3]**

(ii) Hence show that $\displaystyle\int_0^3 \frac{6x^4+12x^3-3x^2-11x-2}{2x^2+4x+1}\,\mathrm{d}x = A + B\ln C$, where A, B and C are constants to be found. **[3]**

6 The equation of a curve is $4\sqrt{y}+x^2y-8=0$. The curve meets the line $y=1$ at two points. Find the gradient of the curve at each of these points. **[7]**

7 The surface of a pond is covered by water lilies. The area of water lilies is denoted by $A\,\mathrm{m}^2$. At $t=0$, $A=10$ and $\dfrac{\mathrm{d}A}{\mathrm{d}t}=0.48$. It is thought that eventually the lilies will cover the whole of the surface area of the pond. A biologist proposes that this situation is modelled by the differential equation

$$\left(\frac{1}{A}+\frac{1}{250-A}\right)\frac{\mathrm{d}A}{\mathrm{d}t}=k$$

where t is the time in days and k is a constant.

(i) Solve this differential equation to express A in terms of t and k. **[6]**

(ii) Find the value of k. **[1]**

(iii) Assuming the model is reliable, find the surface area of the pond. **[1]**

8 **(i)** Given that $y = \ln\left(\frac{1+\sin 4x}{\cos 4x}\right)$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4}{\cos 4x}$. **[4]**

(ii) Find $\int\left(\frac{\cos 2x}{\cos 2x+\sin 2x}+\frac{\sin 2x}{\cos 2x-\sin 2x}\right)\mathrm{d}x$. **[4]**

9 Use the substitution $u = 1+\ln x+x$ to find $\int\frac{3(x+1)(1-\ln x-x)}{x(1+\ln x+x)}\mathrm{d}x$. **[6]**

10 **(i)** Write down a vector equation of the line through the points A (5, 1, 9) and B (8, 7, 15). **[1]**

P is the point (11, −2, 15).

(ii) Show that triangle APB is isosceles and find angle PAB. **[4]**

The point D lies on the line through A and B. Angle PAD = angle PDA.

(iii) Find the coordinates of D. **[4]**

11 The parametric equations of a curve are

$$x = \frac{1}{\sqrt{2+t}} \text{ and } y = t^3-3t \text{ for } -2 < t \leqslant 0.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. **[3]**

(ii) Find the coordinates of the stationary point on the curve and determine its nature. **[4]**

(iii) State the range of values of x and the range of values of y. **[2]**

(iv) Sketch the curve. **[1]**

END OF QUESTION PAPER

Question		Answer	Marks	Guidance	
1	i	$1 + 2x$	**B1**		
		$\left(\frac{1}{4}\right)\times\left(-\frac{3}{4}\right)\times\frac{(8x)^2}{2!}$ oe soi	**M1**	allow bracket error	if **M0** allow **SC1** for $1+4x-8x^2$
		$1 + 2x - 6x^2$ cao	**A1**		ignore extra terms
			[3]		
1	ii	valid for $\lvert x\rvert < \frac{1}{8}$ oe	**B1**		
			[1]		
2		Any two from			
		$3+\lambda=-1-3\mu$ $\lambda=8+\mu$ $2+3\lambda=2-5\mu$	**B1**	may be in vector form	
		solve simultaneously to obtain a value of λ or μ	**M1**		
		$\lambda = 5$ or $\mu = -3$	**A1**		
		(8, 5, 17) isw	**A1**	allow vector form	
			[4]		

Question			Answer	Marks	Guidance	
3			$\frac{1}{4}e^{4x}$ soi	B1	from integration	
			$[16]x \times \frac{1}{4}e^{4x} - \int [16] \times \frac{1}{4}e^{4x} dx$ oe	M1*	allow sign errors only	ignore limits at this stage
			$F[x] = \left[4xe^{4x} - e^{4x}\right]$	A1		
			$F[1] - F[0]$	M1dep*	allow bracket errors, but substitution of limits must be shown	**NB** double negative may be implied by plus sign
			$= 3e^4 + 1$ **NB AG**	A1 [5]	convincing intermediate step needed eg $4e^4 - e^4 - \left(0 - e^0\right)$	no recovery from bracket errors for this mark
4			$\frac{A}{3+x} + \frac{B}{1-x} + \frac{C}{2x+1}$	B1	if not seen here, may be awarded at end	
			$\left[9x^2 + 43x + 8 \equiv\right]$ $A(1-x)(2x+1) + B(3+x)(2x+1) + C(3+x)(1-x)$ soi	M1	allow sign errors only	
			$A = 2$ $B = 5$ $C = -3$ isw	A1 A1 A1 [5]	$\frac{2}{3+x} + \frac{5}{1-x} - \frac{3}{2x+1}$	

Question			Answer	Marks	Guidance	
5	i		$3x^2$ seen in quotient and $\pm 6x^2$ seen as leading term in division	M1	if **M0**, **B2** for quotient and **B1** for remainder	
			quotient is $3x^2 - 3$	A1		the quotient and the remainder may be left embedded; but mark the final answer
			remainder is $x + 1$	A1 [3]		
5	ii		$\int \left(3x^2 - 3 + \frac{x+1}{2x^2+4x+1} \right) \mathrm{d}x$	M1FT	their quadratic quotient and their linear remainder	
			$x^3 - 3x + \frac{1}{4}\ln(2x^2+4x+1)$ cao	A1		
			$18 + \frac{1}{4}\ln 31$ cao	A1 [3]		

Question			Answer	Marks	Guidance	
6			$Ay^{-\frac{1}{2}} \times \frac{dy}{dx}$	M1	A is a constant	
			$Bxy + x^2 \frac{dy}{dx}$	M1	B is a constant	**NB** $\frac{-2xy}{x^2 + 2y^{-\frac{1}{2}}}$
			$4 \times \frac{1}{2} y^{-\frac{1}{2}} \times \frac{dy}{dx} + 2xy + x^2 \frac{dy}{dx} [= 0]$	A1		
			$x = \pm 2$	B1	both values	from $4\sqrt{1} + x^2 \times 1 - 8 = 0$
			substitution of (*their* 2, 1) *or* (*their* –2, 1) following differentiation	M1	may follow incorrect rearrangement	
			at (2, 1) $m = -\frac{2}{3}$	A1		association between point and gradient may be evidenced by substitution
			at (− 2, 1) $m = \frac{2}{3}$	A1		
			Alternatively, marks for differentiation may be awarded as follows	[7]		
			$2x \frac{dx}{dy}$	B1		
			$2x \frac{dx}{dy} y + x^2 \times 1$	M1	use of Product Rule	
			$2x \frac{dx}{dy} y + x^2 + 2y^{-\frac{1}{2}} [= 0]$	A1		

Question			Answer	Marks	Guidance	
7	i		$\ln A - \ln(250 - A) = kt\ (+ c)$	M1*	allow sign error	
			valid substitution of $t = 0$ and $A = 10$ to find c	M1dep*	**NB** $\ln 10 - \ln 240 = 0 + c$	
			$c = -\ln 24$ oe	A1	allow to 3 sf or more	-3.17805383
			constructive log step	A1	eg $\ln\left(\frac{A}{250 - A}\right) = kt - \ln 24$ oe	or $\ln\left(\frac{A}{250 - A}\right) = kt - 3.178$
			taking exponentials correctly of both sides; FT their rearrangement and/or *their* numerical c	M1dep*	eg $\left(\frac{24A}{250 - A}\right) = e^{kt}$	or $\left(\frac{A}{250 - A}\right) = e^{kt-3.178}$
			$[A] = \frac{250e^{kt}}{24 + e^{kt}}$ oe	A1		
			…………………………………………………………..	……….	……………………………………	………………………………………
			Alternatively			
			$\ln A - \ln(250 - A) = kt\ (+ c)$	M1*	allow sign error	
			constructive log step, may be awarded after taking exponentials	A1	eg $\ln\left(\frac{A}{250 - A}\right) = kt + c$	
			taking exponentials correctly of both sides; FT their rearrangement	M1dep*	eg $\frac{A}{250 - A} = e^{kt+c}$	
			valid substitution of $t = 0$ and $A = 10$ to find c	M1dep*	eg $\frac{10}{250 - 10} = e^{0+c}$	
			$\frac{A}{250 - A} = e^{kt-\ln 24}$ oe	A1		
			$[A] = \frac{250e^{kt}}{24 + e^{kt}}$ oe	A1		
				[6]		

Question			Answer	Marks	Guidance	
7	ii		$k = 0.05$	B1 [1]		
7	iii		$A = 250$ [m²]	B1 [1]	ignore commentary	
8	i		$\dfrac{\cos 4x \times 4\cos 4x - (1+\sin 4x)\times -4\sin 4x}{\cos^2 4x}$	M1	quotient rule; allow sign errors and/or one coefficient error	
			$\dfrac{4\cos^2 4x + 4\sin^2 4x + 4\sin 4x}{\cos^2 4x}$ oe	A1		
			$\dfrac{\cos 4x}{1+\sin 4x} \times \textit{their}\, \dfrac{4(1+\sin 4x)}{\cos^2 4x}$	M1	use of chain rule; may be unsimplified	
			$= \dfrac{4}{\cos 4x}$ **NB AG**	A1		
			Alternatively			
			$\dfrac{4\cos 4x}{1+\sin 4x} - \dfrac{-4\sin 4x}{\cos 4x}$	M1	chain rule; allow sign errors and/or one error in coefficient of cos $4x$ or sin$4x$	or use of product rule with (1+sin4x) and (cos4x)$^{-1}$ or sec4x
			$\dfrac{4\cos 4x \times \cos 4x + 4\sin 4x(1+\sin 4x)}{(1+\sin 4x)\cos 4x}$	M1	combine to a single fraction FT *their* chain rule	$(1+\sin 4x)\times -1(\cos 4x)^{-2} \times -4\sin 4x + \dfrac{4\cos 4x}{\cos 4x}$
			eg $\dfrac{4(\cos^2 4x + \sin^2 4x) + 4\sin 4x}{(1+\sin 4x)\cos 4x}$	A1	any equivalent correct step	
			$\dfrac{4}{\cos 4x}$	A1		

Question			Answer	Marks	Guidance	
			Alternatively $\frac{1}{\sec 4x+\tan 4x}\times\left(4\sec 4x\tan 4x+4\sec^2 4x\right)$	**M1**	allow sign errors and/or one coefficient error	
			$\frac{4\sec 4x(\tan 4x+\sec 4x)}{\sec 4x+\tan 4x}$	**M1**	factorising – allow one coefficient slip	
			$4\sec 4x$	**A1**		
			$\frac{4}{\cos 4x}$	**A1**		
				[4]		
8	**ii**		$\frac{\cos 2x(\cos 2x-\sin 2x)+\sin 2x(\cos 2x+\sin 2x)}{(\cos 2x+\sin 2x)(\cos 2x-\sin 2x)}$ oe	**M1**	combine into a single fraction; allow sign errors	allow equivalent form with double angle formulae allow equivalent separate fractions with correct common denominator
			$\frac{\cos^2 2x-\cos 2x\sin 2x+\sin 2x\cos 2x+\sin^2 2x}{\left(\cos^2 2x-\sin^2 2x\right)}$	**A1**	or better	
			$\frac{1}{\cos 4x}$	**A1**		
			$\frac{1}{4}\ln\left(\frac{1+\sin 4x}{\cos 4x}\right)+c$ oe eg $\frac{1}{4}\ln\left(1+\sin 4x\right)+\frac{1}{4}\ln\sec 4x+c$	**A1** **[4]**	**NB** $\frac{1}{4}\ln\left(\sec 4x+\tan 4x\right)+c$	

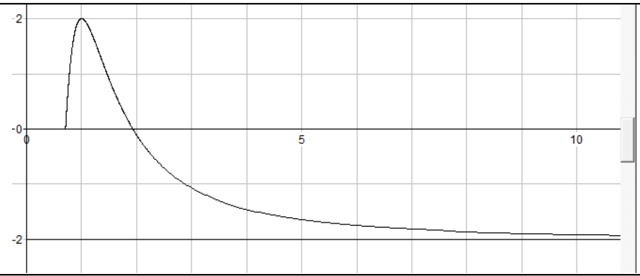
Question			Answer	Marks	Guidance	
9			$\frac{du}{dx} = 1 + \frac{1}{x}$	B1		
			$x + \ln x = \pm u \pm 1$ oe substituted into the numerator	M1*	allow slip in substitution	
			dx replaced by *their* $\left(\frac{1}{1/x + 1}\right)$[d$u$] in integrand oe	M1*		
			$\int\left(\frac{3(1-(u-1))}{u}\right)[du]$ oe	A1	may be simplified	$\int\left(\frac{6}{u} - 3\right)du$
			$A\ln u + Bu$ (+ c)	M1dep*	following $\int\left(\frac{A}{u} + B\right)du$	
			$6\ln(1 + \ln x + x) - 3(1 + \ln x + x) + c$ oe isw	A1 [6]		if du and/or ∫ and/or + c not seen at some stage, withhold the final **A1**
10	i		$r = \begin{pmatrix}5\\1\\9\end{pmatrix} + \lambda\begin{pmatrix}3\\6\\6\end{pmatrix}$ oe isw	B1 [1]	**NB** eg $r = \begin{pmatrix}8\\7\\15\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\2\end{pmatrix}$	**B0** for just the RHS, must see "$r =$" oe
10	ii		$6\times3 - 3\times6 + 6\times6 = \sqrt{6^2+(-3)^2+6^2} \times \sqrt{3^2+6^2+6^2}\cos A$	M1	allow sign errors and 1 algebraic slip eg omission of power	or $\cos A = \frac{9^2+9^2-(their\sqrt{90})^2}{2\times9\times9}$ $PB = 3\sqrt{10}$
			$36 = 81\cos A$ or $-36 = 81\cos A$ or better	A1		
			$A = 63.6°$ or 1.11 rad	A1	if obtuse angle found, clear explanation needed if acute angle stated as answer	$A = 63.6°$ or 1.11 rad
			eg $AB = \sqrt{3^2+6^2+6^2}$ and $AP = \sqrt{6^2+(-3)^2+6^2}$ [so isosceles]	B1 [4]	**NB** $AB = 9$ and $AP = 9$ stated is sufficient **B0** if answer spoiled	**NB** 58.2° or $\cos\theta = \frac{\sqrt{10}}{6}$

Question			Answer	Marks	Guidance	
10	iii		$\overrightarrow{PD}=\begin{pmatrix}5+3\lambda\\1+6\lambda\\9+6\lambda\end{pmatrix}-\begin{pmatrix}11\\-2\\15\end{pmatrix}$ oe	**M1***		NB $\overrightarrow{PD}=\begin{pmatrix}3\lambda-6\\3+6\lambda\\6\lambda-6\end{pmatrix}$
			$(3\lambda-6)^2+(3+6\lambda)^2+(6\lambda-6)^2=9^2$ oe	**M1dep***	allow one algebraic slip eg omission of one power	
			$\lambda=\frac{8}{9}$[or 0]	**A1**	**NB**	
			$\left(\frac{23}{3},\frac{19}{3},\frac{43}{3}\right)$	**A1**	$\lambda=\frac{8}{3}$ if direction vector is $\begin{pmatrix}1\\2\\2\end{pmatrix}$	
				[4]		
			………………………………………………………	……….	………………………………..	
			Alternatively			
			$AD^2=9^2+9^2-2\times9\times9\times\cos(180-2\times63.6)$	**M1***	**NB** $AD=8$	
			$(3\lambda)^2+(6\lambda)^2+(6\lambda)^2=$ their AD^2 oe	**M1dep***		
			$\lambda=\frac{8}{9}$	**A1**	$\lambda=\frac{8}{3}$ if direction vector is $\begin{pmatrix}1\\2\\2\end{pmatrix}$	
			$\left(\frac{23}{3},\frac{19}{3},\frac{43}{3}\right)$	**A1**		
				[4]		

Question			Answer	Marks	Guidance	
10	iii		*Alternatively*			
			$\overrightarrow{PE} = \begin{pmatrix} 5+3\lambda-11 \\ 1+6\lambda--2 \\ 9+6\lambda-15 \end{pmatrix}$	M1	E is the foot of the perpendicular from P to AB	
			$\overrightarrow{PE} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 0$	M1		
			$\lambda = \frac{4}{9}$	A1		
			$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$	A1	from $\overrightarrow{AD} = 2\overrightarrow{AE}$	
				[4]		
			Alternatively			
			$\overrightarrow{PD}$ found as above	M1	eg $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda \\ 6\lambda \\ 6\lambda \end{pmatrix} = \begin{pmatrix} -3\lambda \\ -6\lambda \\ -6\lambda \end{pmatrix} \cdot \begin{pmatrix} 6-3\lambda \\ -3-6\lambda \\ 6-6\lambda \end{pmatrix}$	or $\begin{pmatrix} -2 \\ -6 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 6-3\lambda \\ -3-6\lambda \\ 6-6\lambda \end{pmatrix} = 36$
			$\overrightarrow{AP} \cdot \overrightarrow{AD} = \overrightarrow{DA} \cdot \overrightarrow{DP}$ oe	M1		or $\begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3\lambda \\ 6\lambda \\ 6\lambda \end{pmatrix} = 32$
			$\lambda = \frac{8}{9}$	A1		
			$\left(\frac{23}{3}, \frac{19}{3}, \frac{43}{3}\right)$	A1		
				[4]		

Question			Answer	Marks	Guidance	
11	i		$\left(\frac{dy}{dt}\right) = 3t^2 - 3$	B1		
			$\left(\frac{dx}{dt} =\right) k(2+t)^{-\frac{3}{2}}$	M1	$k \neq 0$	
			$\frac{dy}{dx} = \frac{3t^2 - 3}{-\frac{1}{2}(2+t)^{-\frac{3}{2}}}$ oe isw	A1 [3]	do not allow bracket errors in marked answer	
			Alternatively			
			$[y =](x^{-2} - 2)^3 - 3x^{-2} + 6$ oe	B1		
			$\left[\frac{dy}{dx} =\right] 3(x^{-2} - 2)^2 \times (-2x^{-3}) + 6x^{-3}$ oe	M1	allow sign errors and/or one coefficient error	
			$3\left[\left((2+t)^{-\frac{1}{2}}\right)^{-2} - 2\right]^2 \times -2\left((2+t)^{-\frac{1}{2}}\right)^{-3}$ $+6\left((2+t)^{-\frac{1}{2}}\right)^{-3}$ oe isw	A1 [3]		

Question			Answer	Marks	Guidance	
11	ii		their $\frac{dy}{dx}=0$	M1	allow eg $3t^2-3=0$	allow one transcription error
			(1, 2) oe identified as only stationary point	A1	**NB** $t=-1$	
			eg $t=-0.5$, $x=\sqrt{⅔}$ **and** gradient = 8.27 eg $t=-1.5$, $x=\sqrt{2}$ **and** gradient = -2.65 **or** eg $t=-0.5$ and $y=1.375$, $t=-1.5$ and $y=1.125$	M1	consideration of gradient either side of *their* $x=1$ **or** consideration of y-values either side of their $y=2$	ignore work with other points for the last two marks
			hence maximum value at (1, 2)	A1		
			Alternatively, for last two marks			
			evaluation of second derivative at *their* $t=-1$ or *their* $x=1$	M1		ignore work with other points for the last two marks
			$\frac{d^2y}{dx^2}=-18x^{-4}+24x^{-8}-48x^{-6}+18x^{-4}(x^{-2}-2)^2$ or oe $6(2+t)^2(7t^2+8t-3)$		second derivative must be obtained from correct method; allow sign errors	
			convincing justification that second derivative <0 [NB – 24] so maximum	A1 [4]		
11	iii		$x\geq\frac{1}{\sqrt{2}}$	B1		
			$-2<y\leq 2$	B1 [2]		

Question			Answer	Marks	Guidance	
11	iv			B1 [1]	curve with maximum in 1st quadrant and horizontal asymptote in 4th quadrant drawn for $x \geq k$, where $k > 0$	